

An Incoherent $\alpha - \Omega$ Dynamo in Accretion Disks

Ethan T. Vishniac

Department of Astronomy, University of Texas, Austin TX 78712, USA

I: ethan@astro.as.utexas.edu

Axel Brandenburg

Nordita, Blegdamsvej 17, DK-2100, Copenhagen Ø, Denmark I: brandenb@nordita.dk

ABSTRACT

We use the mean-field dynamo equations to show that an incoherent alpha effect in mirror-symmetric turbulence in a shearing flow can generate a large scale, coherent magnetic field. We illustrate this effect with simulations of a few simple systems. In accretion disks, this process can lead to axisymmetric magnetic domains whose radial and vertical dimensions will be comparable to the disk height. This process may be responsible for observations of dynamo activity seen in simulations of dynamo-generated turbulence involving, for example, the Balbus-Hawley instability. In this case the magnetic field strength will saturate at $\sim (h/r)^2$ times the ambient pressure in real accretion disks. The resultant dimensionless viscosity will be of the same order. In numerical simulations the azimuthal extent of the simulated annulus should be substituted for r . We compare the predictions of this model to numerical simulations previously reported by Brandenburg et al. (1995). In a radiation pressure dominated environment this estimate for viscosity should be reduced by a factor of $(P_{gas}/P_{radiation})^6$ due to magnetic buoyancy.

1. Introduction

Understanding the transport of angular momentum in accretion disks is one of the basic challenges in modern astrophysics. The traditional approach (Shakura, & Sunyaev 1973) is to assume that accretion disks are characterized by an effective viscosity, arising from an unspecified collective process, given by $\alpha_{SS} c_s h$, where c_s is the local sound speed, h is the disk half-thickness, and α_{SS} is a constant of order unity. More recently, there has been the realization (Balbus & Hawley 1991) that a previously discovered magnetic field instability in a shearing flow (Velikhov 1959, Chandrasekhar 1961) will act to produce a

positive angular momentum flux in an accretion disk. This has given rise to two separate, but related claims. The first is the proposal that this is the dominant mechanism of angular momentum transport in ionized accretion disks. The second is the proposal that this instability, by itself, leads to a turbulent dynamo which drives the magnetic field into equipartition with the ambient pressure, i.e. $V_A \sim c_s$, where V_A is the Alfvén speed in the disk. The growth rate for this dynamo is usually taken, following the original claim of Balbus and Hawley, to be $\sim \Omega$. Since the dimensionless ‘viscosity’, α_{SS} , is $\sim (V_A/c_s)^2$, this proposal requires that α_{SS} be a number ‘of order unity’. However, we need to interpret this phrase generously. In numerical simulations (e.g. Brandenburg, Nordlund, Stein, & Torkelsson 1995) α_{SS} is less than 10^{-2} both because the magnetic pressure saturates at a fraction of the gas pressure, and because the off-diagonal components of $\langle \vec{B}\vec{B} \rangle$ are a fraction of B_θ^2 .

Three dimensional simulations of the Balbus-Hawley instability have been performed by a variety of researchers, with and without imposed vertical magnetic flux, and with and without vertical disk structure (Hawley, Gammie, & Balbus 1995a, Brandenburg, Nordlund, Stein, & Torkelsson 1995, Stone, Hawley, Gammie, & Balbus 1995, and Hawley, Gammie, & Balbus 1995b). We note in particular Brandenburg et al. (1995) in which no net flux was imposed on the computational box, and vertical disk structure was included. In this simulation, as in the others, there was an initial rise in the magnetic energy density at a rate $\sim \Omega$. At the end of this phase the system had not yet lost memory of its initial conditions, but after a somewhat longer time, which may be as long as a few dozen rotation periods, the simulation asymptotically approached a final state with $V_A \sim c_s$. The approach to this state was characterized by the appearance of a large scale field which underwent spontaneous reversals at irregular intervals of tens of rotational periods.

Interestingly, the presence of a large scale coherent field does not seem to be due to an $\alpha - \Omega$ dynamo, because the relative helicity is just a few percent. Conventional $\alpha - \Omega$ dynamo models rely on a nonzero $\alpha_{\theta\theta}$ component in the helicity tensor (not to be confused with the dimensionless viscosity, written here as α_{SS}) to produce a large scale coherent field. The presence of an initial rapid rise is less surprising, since imposing a uniform large scale magnetic field in a turbulent medium results in the formation of intermittent magnetic field structures and a consequent rise in the magnetic energy density at the turbulent eddy turn-over rate. In addition, there is evidence (Meneguzzi, Frisch, & Pouquet 1981) that turbulence in a conducting fluid can generate a modest and highly disordered magnetic field even in the absence of an imposed global field. Both of these effects are probably due to the ability of symmetric turbulence to produce a negative effective diffusion coefficient (Moffatt 1978) and they both can be relied open to contribute to the growth of the high wavenumber component of the magnetic field. On the other hand, the slower relaxation rate

seen after the initial rise is correlated with changes in the large scale field and is presumably an indicator of the large scale dynamo growth rate. Since the turbulence is sustained by an instability of the large scale field, its ability to generate such a field is critically important.

The saturation level of the magnetic field in these simulations also leads to some puzzling questions. The claim that the Balbus-Hawley instability saturates when the magnetic pressure is comparable to the ambient thermal pressure, and that the dimensionless viscosity has some approximately fixed value, is difficult to reconcile with attempts to model accretion disks in compact binary systems. Successful models of dwarf novae outbursts and X-ray transients (Smak 1984a, Smak 1984b, Meyer & Meyer-Hofmeister 1984, Huang & Wheeler 1989, Mineshige & Wheeler 1989, and, more recently, Cannizzo 1994), as well as the distribution of light in quiescent dwarf novae disks (Mineshige, & Wood 1989) all imply that the dimensionless viscosity, α_{SS} , varies spatially and with time. These variations are consistent with $\alpha_{SS} \propto (h/r)^n$, where n is a constant lying somewhere between 1 and 2. Recent work (Cannizzo, Chen, & Livio 1995) on X-ray transients suggests that n may be close to 1.5. Here we note only that any value of n appreciably different from zero conflicts with claims for a universal value of α_{SS} .

This difficulty can be resolved in several different ways. For example, we might claim that magnetic instabilities dominate α_{SS} only at low temperatures and that some other process, e.g. convection, dominates at higher temperatures. This idea faces two major objections. First, it explains only some of the phenomenological evidence favoring a varying α_{SS} . Second, attempts to model the vertical structure of dwarf novae disks invariably conclude that such disks are convective during quiescence, when α_{SS} is small and stably stratified during outburst, when α_{SS} is relatively large (for a recent discussion of the conditions necessary for convection in partially ionized accretion disks see Cannizzo 1992). This implies that convection could explain the rise in α_{SS} only if it acts to suppress angular momentum transport, rather than enhance it. Alternatively, one could appeal to the temperature dependence of the resistivity to account for this effect, although the effective resistivity of the simulations is, in any case, many orders of magnitude larger than in real disks. A more promising notion is that one might ascribe the rise in α_{SS} to the greater thermal conductivity of disk in the hot state, although the rationale for this is not yet clear. Finally, one might simply conclude that all the phenomenological models are wrong, for a variety of reasons, a viewpoint which is difficult to dismiss given the large uncertainties faced in modeling accretion disks.

In this paper we will explore a new turbulent disk dynamo in which the turbulence is not assumed to lack mirror symmetry in the vertical direction. The dynamo effect arises from the fact that in a system of finite size the mean square helicity, and the instantaneous

spatially averaged helicity, is still nonzero. We will see that this leads to the existence of a modified $\alpha - \Omega$ dynamo, in which the large scale organization of the magnetic field comes from the existence of a large scale shear. Of course, a real disk has vertical structure, which breaks the vertical symmetry and allows for the *possibility* of a nonzero average helicity. However, we will show that the incoherent dynamo mechanism will be particularly effective in simulations of limited azimuthal extent. Indeed the simulations of Brandenburg et al. (1995) show that while the relative helicity is small (less than a few percent), there is still a dynamo effect leading to the generation of large scale fields. Moreover, the saturation level of the magnetic field, and the consequent value of α_{SS} , turn out to depend sensitively on the ratio h/r . If this dynamo is the only effect to arise from the turbulence induced by the Balbus-Hawley effect, then it is relatively simple to reconcile the dynamo activity seen in simulations with phenomenological models of accretion disks in compact binary systems.

In §2 we discuss the conceptual basis of an incoherent dynamo in a turbulent shearing medium and estimate the growth rate. In §3 we apply this to accretion disks and show that the incoherent dynamo gives a positive growth rate only for axisymmetric magnetic domains. We estimate the saturated state of the field and discuss our results in light of numerical simulations of magnetic fields in a Keplerian disk. In §4 we summarize our results and their implications for astrophysical disks and numerical simulations of such disks.

2. The Incoherent Dynamo

In a highly conducting medium the magnetic field obeys the induction equation

$$\partial_t \vec{B} = \vec{\nabla} \times (\vec{V} \times \vec{B}), \quad (1)$$

where we have neglected ohmic diffusion. Ultimately this term is important in allowing reconnection and smoothing. Here we assume that these processes take place at a rate determined by turbulent processes. The usual approach to dynamo theory is to define the response of the large scale magnetic field to small scale motions as \vec{b} and to derive its effects on the large scale field by substituting \vec{b} back into the right hand side of equation (1). For an incompressible fluid this yields

$$\partial_t B_i = \epsilon_{ijk} \partial_j (\alpha_{kl} B_l) + \partial_j D_{jk} \partial_k B_i - \partial_j D_{ik} \partial_k B_j, \quad (2)$$

where ϵ_{ijk} is the Levi-Civita tensor and

$$\alpha_{kl} \equiv \langle \epsilon_{kij} V_i \partial_l \int^t V_j(t') dt' \rangle, \quad (3)$$

and

$$D_{jk} = \langle V_j \int^t V_k(t') dt' \rangle. \quad (4)$$

The first term comes from the stretching of large scale field lines by the local turbulence. The tensor α_{kl} describes the twisting of large scale field lines into a spiral shape. Reconnection between adjacent spirals produces a large scale field component at right angles to the original field line provided that either the degree of twisting or the large scale magnetic field strength varies in the third direction. The second term is the usual diffusion term, modified by the presence of the third term.

We can see from equation (3) that each component of α_{ij} has either a factor of V_z or ∂_z . If the local velocity field is mirror symmetric, in the sense that its statistical properties are unchanged under the transformation $z \rightarrow -z$, then the time and space averaged value of α_{ij} vanishes. This poses a significant but, as we will see, not insurmountable, obstacle to a successful dynamo.

Another problem is that equations (2), (3) and (4) are usually defined kinematically, i.e. the velocity field is assumed to be imposed on the magnetic field. Once the magnetic field becomes sufficiently powerful it will modify the flow, which is usually taken into account by including a correction term proportional to B^2 . However, in a Keplerian shearing flow the magnetic field will be unstable and the resulting turbulence will be directly correlated with the magnetic field. Nevertheless, as long as we define \vec{V} in terms of the motion of the magnetic field lines equation (2) will remain valid, if difficult to solve. Here we will define our results in terms of the properties of α_{ij} and D_{ij} regardless of their ultimate source.

In a Keplerian disk the dynamo equations can be simplified as

$$\partial_t B_r = -\partial_z(\alpha_{\theta\theta} B_\theta) - \partial_z(V_b B_r) + \partial_z(D_{zz} \partial_z B_r), \quad (5)$$

and

$$\partial_t B_\theta = -\frac{3}{2}\Omega B_r - \partial_z(V_b B_\theta) + \partial_z(D_{zz} \partial_z B_\theta) + \partial_r(D_{rr} \partial_r B_\theta), \quad (6)$$

where $\Omega \propto r^{-3/2}$ is the rotation frequency, V_b is the buoyant velocity of the magnetic field lines relative to the surrounding fluid, and B_r and B_θ are the radial and azimuthal components of the magnetic field. Equations (5) and (6) differ from equation (2) in that we have allowed for the presence of global shearing, and magnetic field line buoyancy. In addition, we have assumed that the diffusion matrix is diagonal, and dropped the effects of helicity on the evolution of B_θ , given that the shearing of B_r should dominate such effects. Also, we have retained only the $\alpha_{\theta\theta}$ term in equation (5) since the critical feedback term in the dynamo equations involves generating radial magnetic flux from the azimuthal component of the field. Finally, given that we are interested in applying these equations to

accretion disks whose thickness is a small fraction of their radius, we have assumed that vertical gradients will dominate over radial gradients.

Now let's assume that the turbulence is symmetric under $z \rightarrow -z$ so that $\langle \alpha_{\theta\theta} \rangle = 0$. Although this eliminates any coherent helicity, the value of $\langle B_r^2 \rangle$ can still increase in a random walk. Ignoring diffusion and buoyancy we see that the formal solution for B_r is

$$B_r = \int^t -\partial_z(\alpha_{\theta\theta}(t')B_\theta(t'))dt'. \quad (7)$$

Now by hypothesis, $\alpha_{\theta\theta}$ is uncorrelated over time scales greater than some eddy correlation time τ_{eddy} . If the radial magnetic field is undergoing a random walk, then it will usually be far enough away from zero that it will not change sign every eddy correlation time. Since B_r drives B_θ through coherent shearing, this implies that the correlation time for B_r and B_θ is much greater than τ_{eddy} . Consequently, we can consider the integrand in equation (7) as consisting of a rapidly varying factor, $\alpha_{\theta\theta}$, multiplying a slowly varying function. Multiplying equation (7) times equation (5) and ignoring diffusion and buoyancy, as before, we see that the integral in equation (7) is correlated with $\alpha_{\theta\theta}$ only over the last eddy correlation time τ_{eddy} . Consequently, we can replace the integral in the product with $-\partial_z(\alpha_{\theta\theta}(t)B_\theta(t))\tau_{eddy}$. This implies

$$\partial_t \langle B_r^2 \rangle \approx K_z^2 \frac{\langle \hat{\alpha}_{\theta\theta}^2 \rangle}{N} \tau_{eddy} \langle B_\theta^2 \rangle, \quad (8)$$

where N is the number of independent turbulent eddies in a magnetic domain, K_z is the vertical wavenumber of the magnetic domain, and $\langle \hat{\alpha}_{\theta\theta}^2 \rangle$ is the mean square helicity associated with a single eddy. In general this will be of order V_T^2 where V_T is the root mean square turbulent velocity. Since B_r is being driven incoherently we can expect it to undergo frequent reversals. In between such reversals the shearing of the field will drive B_θ^2 sharply upward. From equations (6) and (8) we see that the correlation time of the radial magnetic field and the growth time of the magnetic field are comparable and given by

$$\tau_{corr}^{-1} \sim \tau_{growth}^{-1} \sim \left(\frac{K_z^2 V_T^2 \Omega^2 \tau}{N} \right)^{1/3}. \quad (9)$$

We note that τ_{corr} has to be greater than τ_{eddy} in order for this estimate to be internally self-consistent, i.e. the magnetic field must be correlated over longer times than the turbulence itself. Since a field reversal in B_θ requires that B_r not only reverse its sign, but maintain it long enough to push B_θ through zero, it is clear that the correlation time for B_θ may be somewhat larger than the correlation time for B_r . We will return to this point later.

By itself this argument does not show that a succession of random twists in a shearing background can drive an exponential increase in the magnetic field. We need to show that

the growth experienced between field reversals dominates over the abrupt cancellation of the field as B_r reverses itself. We also need to show that our estimate of the growth rate given in equation (9) will dominate over turbulent diffusion for some range of magnetic domain sizes.

We can test the assertion that a series of random changes in B_r can drive a dynamo by constructing a simple toy model of the process, which ignores the spatial structure of the field, but includes its dynamical evolution. Assuming that $\alpha_{\theta\theta}$ has a stochastic component and ignoring buoyancy we can rewrite equations (5) and (6) as

$$\partial_t B_r = (\eta(t) - \alpha_{coh})B_\theta - DB_r, \quad (10)$$

and

$$\partial_t B_\theta = -\frac{3}{2}\Omega B_r - DB_\theta, \quad (11)$$

where $\eta(t)$ is a stochastic variable with a correlation time τ_{eddy} and α_{coh} is the coherent component of $\partial_z \alpha_{\theta\theta}$. Here we have subsumed spatial derivatives into the definitions of η and D and ignored the $-\alpha_{\theta\theta}\partial_z B_\theta$ term which would normally appear in the mean-field dynamo equations. We have also assumed that turbulent damping is the same for each component of the magnetic field, which is not generally true, but simplifies the analysis without losing any essential physics. Equations (10) and (11) can be rewritten in a more convenient form by defining $A \equiv (B_r/B_\theta)$. Then

$$\partial_t A = \eta(t) - \alpha_{coh} + \frac{3}{2}\Omega A^2, \quad (12)$$

and

$$\partial_t \ln B_\theta^2 = -3\Omega A - 2D. \quad (13)$$

The magnetic field will grow exponentially if $\langle A \rangle$ is negative and $-3\Omega\langle A \rangle > 2D$.

We can find $\langle A \rangle$ by solving equation (12) in terms of an unnormalized probability distribution function $P(A)$ and evaluating

$$\langle A \rangle \equiv \frac{\int_{-\infty}^{\infty} AP(A)dA}{\int_{-\infty}^{\infty} P(A)dA}. \quad (14)$$

The distribution function $P(A)$ satisfies the equation

$$\partial_A(\dot{A}P(A) - \langle \eta^2 \rangle \tau_{eddy} \partial_A P(A)) = 0, \quad (15)$$

or

$$P(A) \left(\frac{3}{2}\Omega A^2 - \alpha_{coh} \right) - \langle \eta^2 \rangle \tau_{eddy} \partial_A P(A) = \langle \eta^2 \rangle \tau_{eddy}, \quad (16)$$

where we have taken advantage of the unnormalized nature of $P(A)$ to set the constant of integration to $\langle \eta^2 \rangle \tau_{eddy}$. Equation (16) can be solved to yield

$$P(A) = \exp \left[\frac{\Omega A^3 - 2\alpha_{coh} A}{2\langle \eta^2 \tau_{eddy} \rangle} \right] \int_A^\infty \exp \left[\frac{-\Omega r^3 + 2\alpha_{coh} r}{2\langle \eta^2 \tau_{eddy} \rangle} \right] dr. \quad (17)$$

Consequently,

$$\langle A \rangle = \left(\frac{2\langle \eta^2 \tau_{eddy} \rangle}{\Omega} \right)^{1/3} \frac{\int_{-\infty}^\infty dy \int_y^\infty ds \exp[y^3 - s^3 - \gamma(y - s)]}{\int_{-\infty}^\infty dy \int_y^\infty ds \exp[y^3 - s^3 - \gamma(y - s)]}, \quad (18)$$

where

$$\gamma \equiv \frac{\alpha_{coh}}{\langle \eta^2 \tau_{eddy} \rangle} \left(\frac{2\langle \eta^2 \tau_{eddy} \rangle}{\Omega} \right)^{1/3}. \quad (19)$$

Equation (18) can be rewritten by defining new variables $w \equiv y + s$ and $x \equiv s - y$ and integrating over w . We obtain

$$\langle A \rangle = \frac{-1}{2} \left(\frac{2\langle \eta^2 \tau_{eddy} \rangle}{\Omega} \right)^{1/3} \frac{\int_0^\infty x^{1/2} \exp[x(\gamma - \frac{x^2}{4})] dx}{\int_0^\infty x^{-1/2} \exp[x(\gamma - \frac{x^2}{4})] dx}. \quad (20)$$

When γ is small we can expand $e^{w\gamma} \approx 1 + \gamma w$ and obtain

$$\langle A \rangle \approx -0.32 \left(\frac{\langle \eta^2 \tau_{eddy} \rangle}{\Omega} \right)^{1/3} (1 + 0.51\gamma), \quad (21)$$

which implies that

$$\partial_t \ln B_\theta^2 \approx 0.96 (\langle \eta^2 \tau_{eddy} \rangle \Omega^2)^{1/3} + 0.61 \alpha_{coh} \left(\frac{\Omega}{\langle \eta^2 \tau_{eddy} \rangle} \right)^{1/3} - 2D. \quad (22)$$

In other words, the magnetic field will grow exponentially roughly as fast as the estimate given in equation (9). This will be suppressed by turbulent diffusion only when the damping rate due to diffusion is comparable to the growth rate.

The existence of an incoherent dynamo emerges from the fact that the distribution function $P(A)$ given in equation (17) is biased towards negative values of A . This bias comes, paradoxically enough, from the coherent, positive definite term in equation (12). When A is sufficiently positive it evolves deterministically through $+\infty$ into negative values. (Actually, B_r doesn't change during this phase. This deterministic trajectory is merely a field reversal for B_θ .) The end result is that whenever A becomes large and positive it rapidly switches to being large and negative. Ultimately, the sign of the bias is determined by the sign of $\partial_r \Omega$. The frequency of such field reversals is given by examining the probability distribution at large A when the evolution of $P(A)$ is deterministic. If we

define $\tau(A)$ as the time it takes for the field to move from some large positive value of A to $A = \infty$ then from equation (12) we see that

$$\tau(A)^{-1} = \frac{3}{2}\Omega A, \quad (23)$$

where we have neglected α_{coh} since for A sufficiently large its effects can be ignored. The field reversal rate is just the limit of this rate times the statistical weight of the distribution between A and ∞ . In other words,

$$\tau_{rev}^{-1} = \lim_{A \rightarrow \infty} \tau(A)^{-1} \frac{\int_A^\infty P(s) ds}{\int_{-\infty}^\infty P(s) ds}. \quad (24)$$

Substituting equations (17) and (23) into this result, and making the change of variables, as before, to x and w we have

$$\tau_{rev}^{-1} = \lim_{A \rightarrow \infty} \frac{3}{2}\Omega A \frac{\int_{2A(\Omega/2\langle\eta^2\tau_{eddy}\rangle)^{1/3}}^\infty \int_0^\infty \exp\left[w\left(\gamma - \frac{1}{4}(w^2 + 3x^2)\right)\right] dw dx}{\int_{-\infty}^\infty \int_0^\infty \exp\left[w\left(\gamma - \frac{1}{4}(w^2 + 3x^2)\right)\right] dw dx}. \quad (25)$$

Both the numerator and the denominator can be simplified by integrating over x to obtain

$$\tau_{rev}^{-1} = \left(\frac{3}{\pi}\right)^{1/2} \frac{(2\langle\eta^2\tau_{eddy}\rangle\Omega^2)^{1/3}}{\int_0^\infty w^{-1/2} \exp[w(\gamma - w^2/4)] dw}. \quad (26)$$

When γ is small this becomes

$$\tau_{rev}^{-1} = 0.53(\langle\eta^2\tau_{eddy}\rangle\Omega^2)^{1/3}(1 - 0.53\gamma), \quad (27)$$

i.e. a rate which is roughly half the e-folding rate for the magnetic field energy.

When α_{coh} is large and positive we can evaluate the integrals in equation (20) by expanding around the maximum of $x(\gamma - x^2/r)$. We obtain

$$\langle A \rangle \approx - \left(\frac{2\alpha_{coh}}{3\Omega} \right)^{1/2}, \quad (28)$$

so that

$$\partial_t \ln B_\theta^2 \approx 2 \left(\frac{3}{2}\alpha_{coh}\Omega \right)^{1/2} - 2D, \quad (29)$$

which is the expected result for a coherent $\alpha - \Omega$ dynamo. In this limit the magnetic field reversal rate becomes

$$\tau_{rev}^{-1} = 0.68\alpha_{coh}^{1/4} \langle\eta^2\tau_{eddy}\rangle^{1/6} \Omega^{7/12} \exp \left[\frac{-1.1\alpha_{coh}}{\langle\eta^2\tau_{eddy}\rangle^{2/3} \Omega^{1/3}} \right]. \quad (30)$$

As expected, field reversals are exponentially suppressed as we go to the usual $\alpha - \Omega$ dynamo. Given $\alpha_{coh} > 0$ then as α_{coh} becomes significant we expect it to enhance the dynamo growth rate and reduce the rate of spontaneous field reversals.

When α_{coh} is large and negative we can evaluate equation (20) by integrating the denominator by parts and remembering that the bulk of the contribution to the integral comes from $w < -1/\gamma$ so that $w^2 \ll -\gamma$. We obtain

$$\langle A \rangle \approx \frac{\langle \eta^2 \tau_{eddy} \rangle}{4\alpha_{coh}}, \quad (31)$$

and

$$\partial_t \ln B_\theta^2 \approx \frac{-3\Omega \langle \eta^2 \tau_{eddy} \rangle}{4\alpha_{coh}} - 2D. \quad (32)$$

In this limit field reversals occur at a rate given by

$$\tau_{rev}^{-1} = 0.78(-\alpha_{coh}\Omega)^{1/2}. \quad (33)$$

We note that in this case the coherent component of the helicity does not completely shut off the incoherent dynamo, even though by itself it is incapable of driving a dynamo. Instead we find that as $|\gamma|$ increases past one the dynamo growth rate decreases inversely with $|\gamma|$. Eventually, turbulent diffusion will suppress the dynamo. In the limit where γ is of order -1 we anticipate that the dynamo growth rate will be less than expected from the incoherent dynamo alone and the rate of field reversals will be larger.

These analytic results have the advantage of being based on a solvable model, but do not include the effects of spatial structure or saturation. It is therefore instructive to consider the combined effects of a random electromotive force and shear in a one dimensional model. We consider the mean field equations for a uniform disk with Keplerian rotation and half-thickness H ,

$$\partial_t B_r = -\partial_z(\alpha B_\theta) + D_t \partial_z^2 B_r, \quad (34)$$

$$\partial_t B_\theta = -\frac{3}{2}\Omega B_r + D_t \partial_z^2 B_\theta, \quad (35)$$

with $-H \leq z \leq H$ and $B_r = B_\theta = 0$ at $z = \pm H$. We first consider the incoherent α -effect, so we take α to be random in space and time. When the rms value of α is large enough we find self-excited solutions that grow without bound. In reality there must be some quenching mechanism, which we model using

$$\alpha = \alpha_0 \frac{\mathcal{N}(z, t)}{(1 + B_\theta^2)}, \quad (36)$$

where \mathcal{N} is a random function in space and time with zero mean and an rms value of unity. Without loss of generality we put $H = \Omega = D_t = 1$.

In fig. 1 we plot contours of the B_θ field in a space-time diagram for a dynamo number $\alpha_0 \Omega H^3 / D_t^2$ of 10^4 . (The critical dynamo number for dynamo action depends on the coherence time and length scales, λ and τ , respectively. In the present case we adopt $\lambda = 0.05$ and $\tau = 0.002$ and find the critical dynamo number to be around 2000. At this dynamo number the ratio of the growth rate given in equation (9) to D_t/H^2 is ~ 7 .) The remarkable result is that the B_θ field shows a great deal of spatio-temporal coherence with variations comparable to the diffusion time and diffusion length. Experiments with different dynamo numbers suggest that the degree of coherence is more pronounced for larger dynamo numbers.

In order to isolate the effect of a spatially incoherent α -effect we now investigate a model with a steady α -effect of the form

$$\alpha = \alpha_0 \sin(n\pi z). \quad (37)$$

For large values of n , the critical dynamo number is proportional to n^2 . Thus, although the rms value of the α -effect is unchanged, the dynamo becomes harder to excite if α is chopped into many domains of different sign. The magnetic field is steady, and the radial component is of alternating sign. However, more surprisingly, the toroidal magnetic field has the same sign for all values of z , see fig. 2. This is very similar to the simulation of a random incoherent α -effect mentioned before. There is one difference in that the magnetic field shows global reversals in time when the α -effect is incoherent in time.

Finally, we note that in order for the magnetic field to grow the growth rate given in equation (9) has to be greater than the dissipation rate. In general the dissipation rate will depend on the wavenumber of the magnetic domain as K^2 , while the growth rate goes as $(K^2/N)^{1/3}$. Clearly whether or not there is a self-excited dynamo will depend in large part on the geometry of the fluid.

3. The Incoherent Dynamo in Accretion Disks

A Keplerian accretion disk with a root mean square Alfvén speed of V_A will be subject to a local instability first described by Velikhov (1959). Its pivotal role in transporting angular momentum outward in accretion disks was recognized later (Balbus & Hawley 1991). In the context of accretion disks this instability is normally referred to as the Balbus-Hawley

instability. Its maximum growth rate is of order Ω , and occurs at an azimuthal wavelength of $\sim V_A/\Omega$. In three dimensions the instability saturates in turbulence with a typical turbulent velocity comparable to V_A and a typical eddy size of $\sim V_A/\Omega$. This turbulence is not expected to be isotropic, but the typical eddies are expected to have axis ratios of order unity, which in this context means only that no axis should be more than an order of magnitude larger than another (Vishniac, & Diamond 1992). Numerical simulations (Brandenburg, Nordlund, Stein, & Torkelsson 1995) indicate that the azimuthal scale of the typical eddies is several times the vertical and radial scales, which is expected in light of the large local shear. The azimuthal velocity is also larger, although only by a factor of roughly two. Neglecting such factors, these scaling laws imply a turbulent diffusivity of $\sim V_A^2/\Omega$. The turbulence largely suppresses the Parker instability and the typical buoyant velocity of the magnetic field is of order V_A^2/c_s (Vishniac, & Diamond 1992, Vishniac 1995b), where c_s is the local sound speed. The angular momentum flux induced by the turbulence is approximately $\langle V_\theta V_r \rangle \sim V_A^2$ which implies a dimensionless viscosity α_{SS} of order $(V_A/c_s)^2$. Since $h\Omega \sim c_s$ this implies that magnetic flux is lost from the disk at a rate which is some fraction of order unity times $\alpha_{SS}\Omega$.

It is by no means obvious that in real disks this turbulence possesses the kind of symmetry that would make $\langle \alpha_{\theta\theta} \rangle = 0$. On the other hand, calculations done without vertical structure or any imposed large scale field (Hawley, Gammie, & Balbus 1995b) give results which are qualitatively similar to calculations which include vertical structure (Brandenburg, Nordlund, Stein, & Torkelsson 1995). By construction the former calculations must be symmetric under the transformation $z \rightarrow -z$ even though the latter are not. We can in principle estimate $\alpha_{\theta\theta}$ using data from the simulation of Brandenburg et al. (1995). From equation (3) it is clear that a time integration has to be carried out. However, video animations of those data suggest that the life time of turbulent eddies is shorter than the life time of magnetic structures which, in turn, is shorter than the eddy turn over time. In other words, the Strouhal number (e.g. Krause & Rädler 1980) is small. As a rough approximation we may therefore replace the time integration by a multiplication with a relevant time scale. We adopt the natural time scale Ω^{-1} , which is sufficient since we are only interested in relative variations. We adopt volume averages and note that because of the periodic boundary conditions in the toroidal direction, $\langle V_r V_{z,\theta} \rangle = -\langle V_z V_{r,\theta} \rangle$, so we can compute

$$\alpha_{\theta\theta} \approx \frac{2}{r} \langle V_r V_{z,\theta} \rangle \Omega^{-1}. \quad (38)$$

In fig. 3 we plot the evolution of $\alpha_{\theta\theta}$ using the data from run C of Brandenburg et al. (1995), which has now been carried out for an additional 200 orbits, see also Torkelsson et al. (1996). This average was computed for the upper half plane of the simulation. We note that $\alpha_{\theta\theta}$ is positive, in agreement with the expected effect for bubbles that expand as

they rise in a Keplerian disk. However, the sign of $\alpha_{\theta\theta}$ suggested by the correlation between the azimuthal magnetic and electric fields is negative (Brandenburg, Nordlund, Stein, & Torkelsson 1995). The source of this discrepancy is not yet clear. In any case the spatially averaged helicity shows large variations from its long term average, although the variations in the electromotive force are much larger.

The size of the fluctuations in the electromotive force, as well as the persistence of the dynamo in the absence of any \hat{z} symmetry breaking, implies that any preferred helicity resulting from vertical structure is not strong enough to completely dominate the simulations. In what follows we will assume that real disks lack any significant $\langle\alpha_{\theta\theta}\rangle$. At a minimum our results can be taken as demonstrating that there is an incoherent dynamo operating in the simulations, and in real accretion disks, whose effects need to be understood, and cleanly separated from any other dynamo mechanisms at work.

Let's consider a magnetic domain characterized by the wavenumbers (K_r, K_θ, K_z) . Ignoring the anisotropies in the turbulence we find that the number of turbulent eddies per domain is roughly $\sim (K_r K_\theta K_z V_A^3)^{-1} \Omega^3$. Consequently the growth rate for the dynamo is

$$\tau_{dynamo}^{-1} \sim \left(\frac{V_A^5 K_z^3 K_r K_\theta}{\Omega^2} \right)^{1/3}. \quad (39)$$

However, in a shearing environment we aren't free to specify K_r and K_θ separately. The shear implies a minimum K_r for any K_θ since in a time $\sim \tau_{dynamo}$ the shear will increase K_r by an amount $(3/2)K_\theta\Omega\tau_{dynamo}$. If we choose a value of K_r above this minimal value then τ_{dynamo} will go as $K_r^{1/3}$ while the dissipation rate scales as K_r^2 . Clearly our chances for a successful dynamo will be maximized by taking $K_r \sim K_\theta\Omega\tau_{dynamo}$. This gives us

$$\tau_{dynamo}^{-1} \sim \left(\frac{V_A^5 K_z^3 K_r^2}{\Omega^3} \right)^{1/2}. \quad (40)$$

This analysis only makes sense in the limit where the magnetic domains encompass at least one eddy, or $K_z V_A < \Omega$ and $K_r V_A < \Omega$. The dissipation rate is roughly

$$\tau_{dissipation}^{-1} \approx (K_z^2 + K_r^2) \frac{V_A^2}{\Omega}. \quad (41)$$

By comparing equations (40) and (41) we see that the incoherent dynamo is incapable of generating non-axisymmetric large scale magnetic fields. The dissipation rate of such domains exceeds the generation rate for all domain sizes greater than a single eddy.

In a real disk the number of eddies in a magnetic domain does not increase indefinitely as $K_\theta \rightarrow 0$. The finite circumference of the disk implies that for axisymmetric domains

$$N \sim \frac{r\Omega^3}{K_z K_r V_A^3}. \quad (42)$$

Consequently, we can rewrite equation(39) as

$$\tau_{dynamo}^{-1} \sim \left(\frac{V_A^5 K_z^3 K_r}{r \Omega^2} \right)^{1/3}. \quad (43)$$

At a fixed wavenumber, and therefore at a fixed dissipation rate, this rate is maximized for $K_z = K_r 3^{1/2}$. Assuming this ratio we see that the dynamo growth rate for axisymmetric domains goes as $K^{4/3}$, which implies that at some sufficiently small K the dynamo will work. More exactly, the incoherent dynamo caused by the Balbus-Hawley instability will drive an increase in the magnetic field strength if

$$K^2 < \frac{\Omega}{r V_A}. \quad (44)$$

In other words, the incoherent dynamo only works for

$$V_A < \frac{\Omega}{r K^2}. \quad (45)$$

Ultimately K_z is limited by the height of the disk, i.e. $K_z h > 1$. Moreover, as we approach this limit the buoyant loss of magnetic flux becomes significant. The buoyant loss rate from a single magnetic domain goes as

$$\tau_{buoyant}^{-1} \sim K_z V_b \sim K_z \frac{V_A^2}{c_s}, \quad (46)$$

so when $K_z h \sim 1$ buoyant losses are as important as turbulent diffusion. Of course, the only limit on the radial extent of a magnetic domain is $K_r r > 1$, but lowering K_r past h^{-1} will lower the growth rate without affecting the dissipation rate. From equation (45) we see that the magnetic field associated with scales of order the disk thickness will be the strongest and will be given by

$$V_A \sim c_s \frac{h}{r}. \quad (47)$$

This in turn implies that the dimensionless viscosity associated with this dynamo mechanism is

$$\alpha_{SS} \sim \left(\frac{V_A}{c_s} \right)^2 \sim \left(\frac{h}{r} \right)^2. \quad (48)$$

We expect this scaling law to hold only in the limit $h \ll r$. As $V_A \rightarrow c_s$ corrections of order (V_A/c_s) will become important in our formula for buoyancy. Since the saturation limit for the magnetic field involves the small difference between the growth rate dependence on V_A , which has an exponent of 5/3, and the buoyant loss rate dependence, which goes as V_A^2 , we expect the saturation strength of the magnetic field to be extremely sensitive to such

corrections unless $V_A \ll c_s$. We also note that the disk radius enters into this result only through its role as the circumference of an annulus. Computer simulations typically involve a short arc in place of a full annulus. In this case the azimuthal length of the simulation has to be used in place of r in equation (48).

The rate of spontaneous magnetic reversals expected in the absence of any coherent component to $\alpha_{\theta\theta}$ is comparable to dynamo growth rate. However, while current simulations seem to show a significant reversal rate in the presence of vertical structure (Brandenburg, Nordlund, Stein, & Torkelsson 1995), in its absence the field can evolve for 100 orbital times without reversing (Torkelsson, Brandenburg, Nordlund, & Stein 1996). The exact relationship between the dynamo growth rate and the field reversal rate dependent on the particular model for the process, and the zero dimensional model used in this paper may well overestimate the rate of spontaneous field reversals. Nevertheless, such reversals are an intrinsic part of the model, and should occur if the simulation is run for several growth times. The sharp rise in the field reversal rate when vertical structure is included suggests that a significant coherent $\alpha_{\theta\theta}$ is present in the such simulations. (In the zero dimensional model this would argue for a negative vertical gradient in the coherent $\alpha_{\theta\theta}$. The situation is less clear in three spatial dimensions.) In order for this helicity to allow the buildup of a coherent field in these simulations, as well as in accretion disks, it has to scale with the local rms turbulent velocity more steeply than the square of the incoherent dynamo growth rate, or $(V_T/c_s)^{10/3}$. If this helicity is due to the Parker instability and if, as has been argued elsewhere (Vishniac, & Diamond 1992), the Balbus-Hawley instability reduces the Parker instability to vertical motions of order V_T^2/c_s , then we can estimate the magnitude of the helicity as

$$V_r k_\theta V_z \tau_{buoyant}, \quad (49)$$

where $\tau_{buoyant}$ is the correlation time for these buoyant motions. Since shearing imposes the requirement that $k_\theta \Omega < k_r \tau_{buoyant}^{-1}$ and since these motions are approximately incompressible, i.e. $k_r V_r \sim k_z V_z$, this gives a helicity less than

$$V_z^2 k_z / \Omega \sim \frac{V_T^4}{c_s^3}. \quad (50)$$

If the coherent helicity has this dependence, then it becomes important only as the dynamo saturates due to turbulent mixing and buoyancy. In this case it will not suppress the incoherent dynamo in simulations with smaller (h/r) , or in real accretion disks, but it will remain significant in the saturated state.

The fact that the buoyancy does not significantly enhance the loss of magnetic flux is a critical element in the derivation of equation (48). Consequently, environments that increase magnetic buoyancy will saturate at much lower field strengths. As an example we

can consider magnetic flux tubes in a radiation pressure dominated environment. In this case we have (Vishniac 1995b)

$$V_b \sim \frac{P_{\text{radiation}}}{P_{\text{gas}}} \frac{V_A^2}{c_s}. \quad (51)$$

Combining this result with equation (43) for $K_r \sim K_z \sim h^{-1}$ yields

$$\alpha_{SS} \sim \left(\frac{V_A}{c_s}\right)^2 \sim \left(\frac{P_{\text{gas}}}{P_{\text{radiation}}}\right)^6 \left(\frac{h}{r}\right)^2. \quad (52)$$

The exponent given in equation (52) is perhaps a bit large in comparison to the value suggested by the phenomenology of disks, but this is a considerably less serious problem than if it were too small. Competing dynamo mechanisms and/or hydrodynamic angular momentum transport mechanisms could be driving α_{SS} up. How does this compare to other sources of viscosity in disks? The result given in equation (52) has an extremely uncertain coefficient. Current numerical simulations give α_{SS} of order 10^{-2} or less, which would suggest that this coefficient is very small. On the other hand, these simulations have $V_A \sim c_s$ and are definitely not in the asymptotic regime where our scaling laws should be valid. We have already noted that the small difference in the exponent of V_A in the dynamo growth rate and the dissipation rate, coupled to the presence of corrections to both these rates of order (V_A/c_s) , makes it difficult to extrapolate from current results. If the saturation value of V_A/c_s approaches its asymptotic dependence on h/r gradually as $h/r \rightarrow 0$ then the final value of the coefficient will be much larger than 10^{-2} . Bearing in mind the large role that numerical viscosity plays in the simulations (Vishniac 1995a) it seems prudent to regard the coefficient as an unknown numerical constant.

On the other hand, since $h \ll r$ for many realistic disks we can compare this dynamo mechanism to others based purely on the value of the exponent in the scaling relationship. Of course, a purely local mechanism *coherent* mechanism will not scale with (h/r) at all, although it might show some dependence on the local disk temperature. However, as we noted earlier this model seems to conflict with phenomenological studies of dwarf novae and x-ray transients. Internal waves, excited by tidal instabilities in binary system disks (Goodman 1993) will produce an effective α_{SS} which scales as $(h/r)^2$ (Vishniac, & Diamond 1989). This will be a competing mechanism for angular momentum transport in gas pressure dominated disks, and potentially the dominant one in radiation pressure dominated disks. (Although, such conditions are most likely in AGN disks, where the potential for the tidal excitation of waves is less certain.) Given the nonlocal nature of the angular momentum transport mediated by internal waves, the existence of a purely local mechanism might be important, even if it does not clearly dominate. When the disk is ionized and when internal waves are present, then the waves are capable of driving a dynamo with a growth rate

$\sim (h/r)^{3/2}\Omega$ (Vishniac, Jin & Diamond 1990) and perhaps faster, depending on the nature of the turbulent cascade of wave energy (Vishniac, & Diamond 1992). The resulting value of α_{SS} will be $\sim (h/r)^{3/2}(P_{gas}/P)$ (Vishniac, & Diamond 1992, Vishniac 1995b). When these conditions are met this would appear to be a more important dynamo mechanism, although once again we note that nonlocal effects on the wave-driven dynamo make the two processes somewhat incommensurate. An equivalent estimate based on purely local physics was given by Meyer & Meyer-Hoffmeister (1983). However, this estimate is based on using large scale buoyant cells driven by magnetic buoyancy, a picture which is inconsistent with turbulence in the disk (Zweibel & Kulsrud 1975, Vishniac, & Diamond 1992). In addition, they assumed approximate isotropy of the helicity tensor and offered a calculation of α_{rr} instead of $\alpha_{\theta\theta}$. This assumption of isotropy is inconsistent with the notion that the motions are driven by magnetic buoyancy, which for $h \ll r$ will have a time scale much longer than the local shearing time scale. Finally, we note that our result for the incoherent dynamo in an accretion disk is sensitive to our assumption that the process must be self-exciting. Given an external source of fluid turbulence, e.g. convection or the turbulent cascade of energy set off by finite-amplitude internal waves, the incoherent dynamo may have a significantly larger growth rate and give rise to a larger α_{SS} .

4. Conclusions

In this paper we have shown that mean-field dynamo theory allows for the existence of a new kind of $\alpha - \Omega$ dynamo, which we have named the incoherent $\alpha - \Omega$ dynamo, in which there is no coherent helicity whatsoever. In this class of dynamos the magnetic field is driven by a combination of a random walk for B_r and its shearing, which creates B_θ . The resultant large scale field derives its organization from coherent shearing effects, rather than any loss of mirror symmetry in the turbulence. Although this kind of dynamo necessarily includes spontaneous field reversals, such reversals may occur at a rate which is some fraction of the dynamo growth rate. The existence of a mean-field dynamo in a flow with a mean helicity of zero is interesting for its own sake, since it provides an example of how large scale order in the magnetic field can arise from the interaction between a large scale shear and statistically symmetric local motions. In this sense it represents an alternative to models which seek to explain dynamo activity through asymmetric turbulence and a coherent helicity. It differs from previous attempts to do without a coherent helicity (e.g. Montgomery et al. (1984) and Gilbert et al. (1988)) in that it does so without appealing to the other terms in equation (2). This model is particularly interesting in light of previous

claims (Moffatt 1979) that the coherent α effect does not converge (although see Kraichnan (1979) for a counterargument).

This dynamo is particularly interesting in light of simulations of magnetic field instabilities in accretion disks. We have suggested that this dynamo can operate successfully in accretion disks, but only to produce axisymmetric large scale fields. Comparing the growth rate for this dynamo with the buoyant loss rate for magnetic flux, we see that *if* this is the only dynamo associated with magnetic shearing instabilities, then the large scale magnetic field will saturate when $V_A \sim (h/r)c_s$ and $\alpha_{SS} \sim (H/r)^2$. This result may seem somewhat odd, since the dynamic equations do not depend on r at all. However, the factor of r comes in through geometrical considerations, i.e. from considering the number of independent eddies in an axisymmetric magnetic field domain. In that sense it refers to the circumference of such an annulus rather than its radius. Consequently, when comparing numerical simulations to the predictions of this model one should substitute the azimuthal extent of the simulations for r . For current simulations this gives $(h/r) \sim (2\pi)^{-1}$. In fact, since the turbulent eddies are longer in the azimuthal direction, the number of independent eddies that can be stacked end to end in current simulations is one or two, so that effectively $(h/r) \sim 1$. A critical test of this model would be to run simulations where this ratio was small and look for a drop in V_A/c_s in the saturated state. It's interesting to note that the only numerical simulation with no imposed field and with disk vertical structure does seem to have a coherent component to the helicity, which may have the wrong sign to drive a conventional dynamo. This may account for the large rate of spontaneous field reversals when vertical structure is put in the simulations. The existence of such a component is consistent with the existence of an incoherent $\alpha - \Omega$ dynamo, but only if its amplitude scales steeply with the strength of turbulence in the disk.

Finally, we note that this model successfully reconciles phenomenological models of stellar accretion disks and the existence of a dynamo effect in a magnetized disk. The only drawback is that this model gives a relationship between α_{SS} and (h/r) which is probably too steep, implying the existence of other, more efficient dynamo mechanisms in accretion disks in binary systems.

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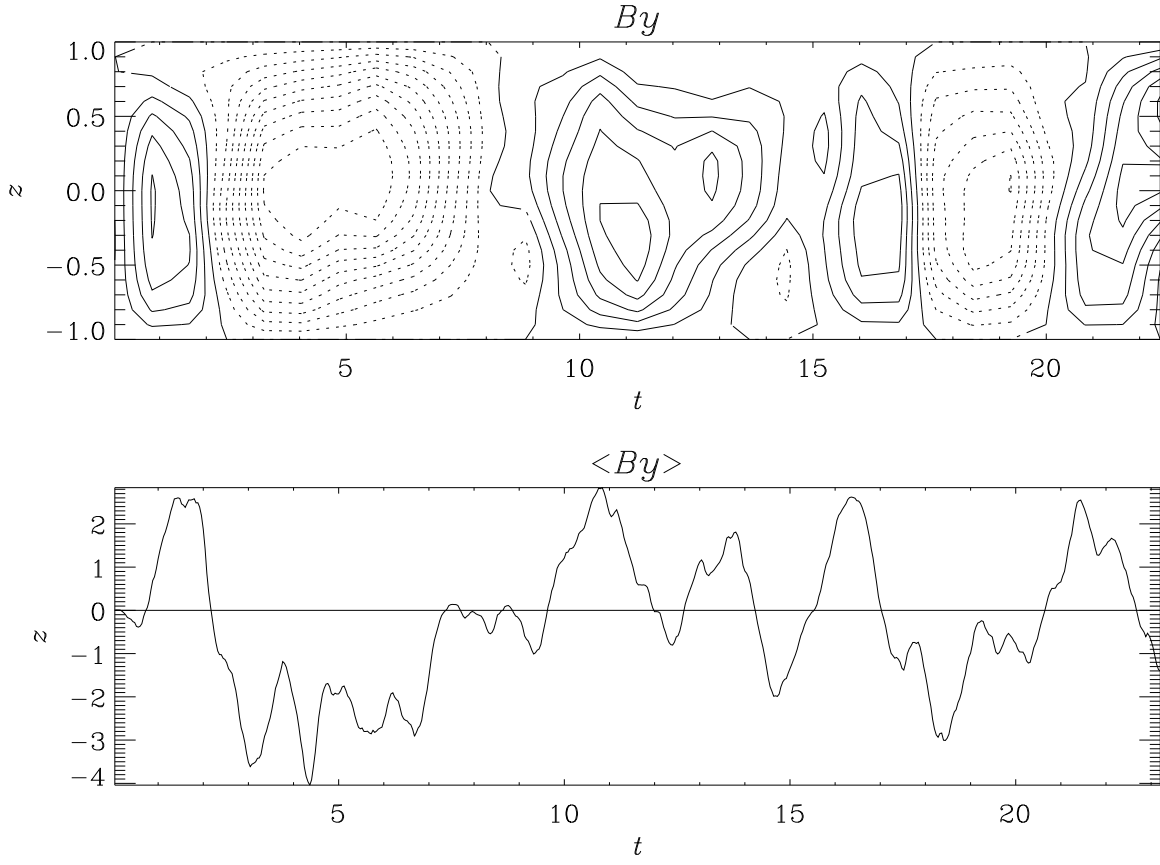


Fig. 1.— Contours of the B_θ field in a space-time diagram for the one dimensional spatially and temporally incoherent dynamo model.

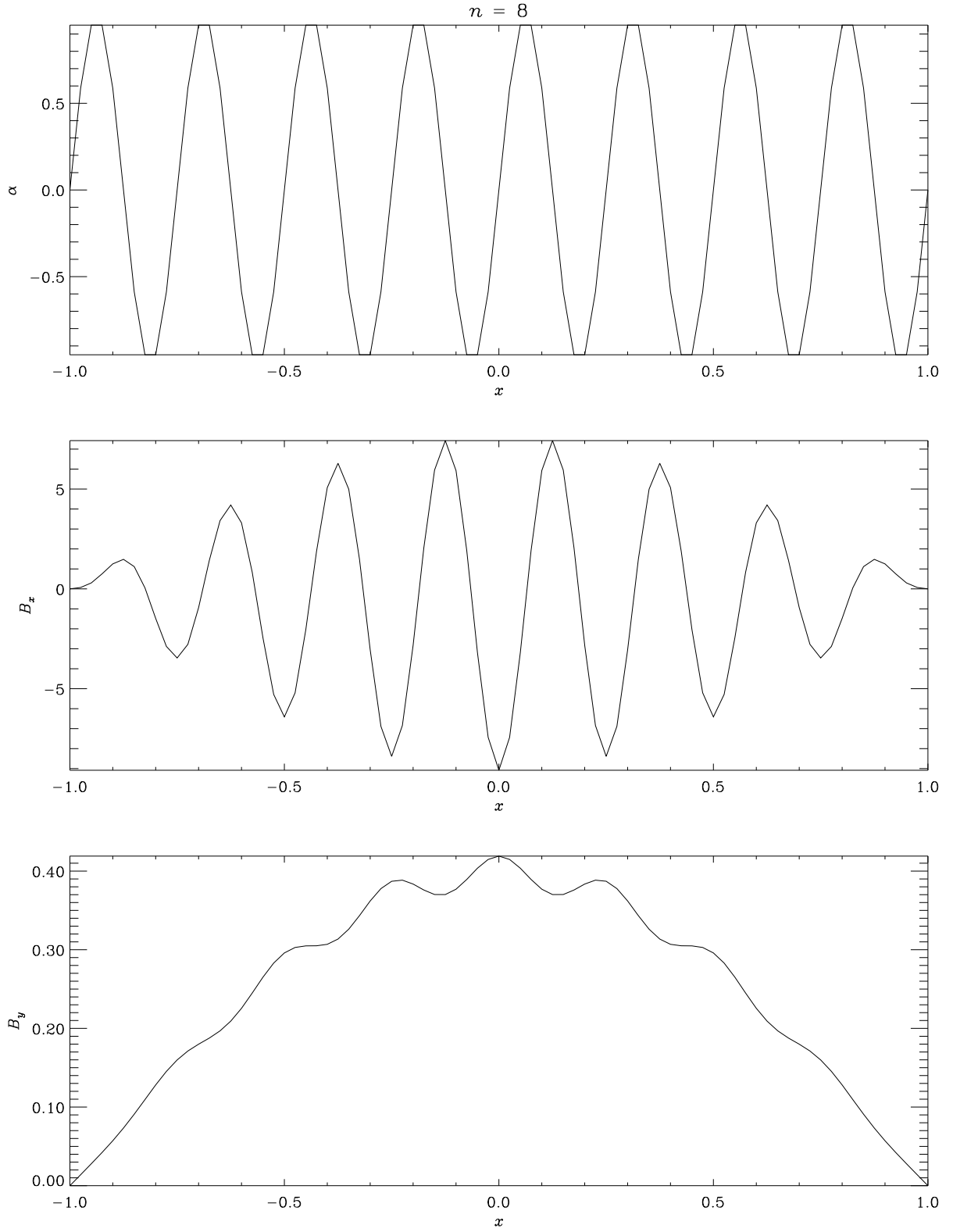


Fig. 2.— A snapshot of the magnetic field and helicity for the one dimensional spatially incoherent dynamo model.

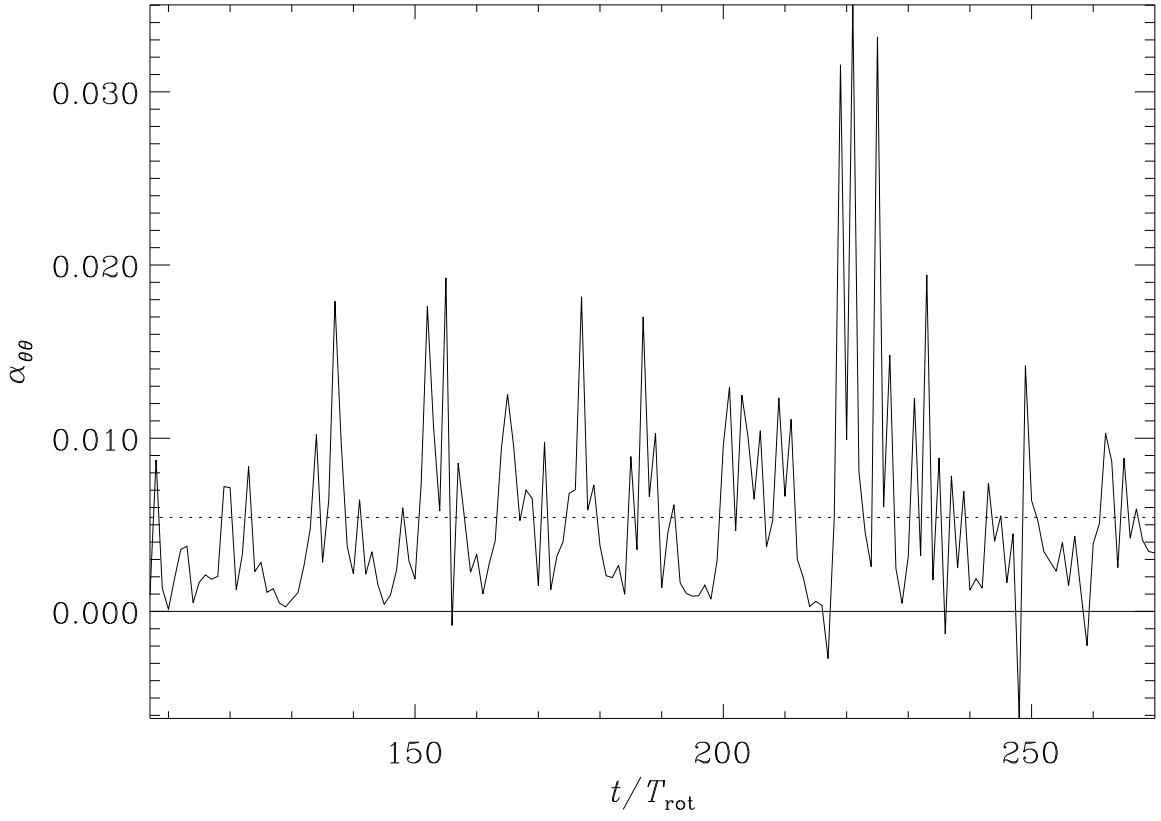


Fig. 3.— The evolution of $\alpha_{\theta\theta}$ (normalized by the product of the rms velocity and $(\Omega\tau_{\text{eddy}})^{-1}$) in the upper half plane of run C from Brandenburg et al. (1995).